Electromagnetics

For

Electronics & Communication Engineering

By

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Syllabus for Electromagnetics

Electrostatics, Maxwell's Equations, Differential and Integral forms and their Interpretation, Boundary Conditions, wave Equation, Pointing Vector, Plane Waves and Properties, Reflection and Refraction, Polarization, Phase and Group Velocity, Propagation Through Various Media, Skin Depth, Transmission Lines, Equations, Characteristic Impedance, Impedance Matching, Impedance Transformation, S-parameters, Smith Chart, Waveguides, Modes, Boundary Conditions, Cut-Off Frequencies, Dispersion Relations, Antennas, Antenna Types, Radiation Pattern, Gain and Directivity, Return Loss, Antenna Arrays, Basics of Radar, Light Propagation in Optical Fibers.

Previous Year GATE Papers and Analysis

GATE Papers with answer key

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Subject wise Weightage Analysis

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"Picture yourself vividly as winning and that alone will contribute immeasurably to success."

...**Harry Fosdick**

CHAPTER 1 Electromagnetic Field

Learning Objectives

After reading this chapter, you will know:

1. Elements of Vector Calculus
2. Operators, Curl, Divergence
3. Electromagnetic Coulombs' law, Electric Field Intensity, Electric Dipole, Electric Flux Density
4. Gauss's Law, Electric Potential
5. Divergence of Current Density and Relaxation
6. Boundary Conditions
7. Biot-Savart's Law, Ampere Circuit Law, Continuity Equation
8. Magnetic Vector Potential, Energy Density of Electric & Magnetic Fields, Stored Energy in Inductance
9. Faraday's Law, Motional EMF, Induced EMF Approach
10. Maxwell's Equations

Introduction

Cartesian coordinates (x, y, z), $-\infty < x < \infty, -\infty < y < \infty, -\infty < z < \infty$

Cylindrical coordinates (ρ , ϕ, z), $0 \le \rho < \infty, 0 \le \phi < 2\pi, -\infty < z < \infty$

Spherical coordinates (r, θ, ϕ) , $0 \le r < \infty, 0 \le \theta \le \pi, 0 \le \phi < 2\pi$

Other valid alternative range of θ and ϕ are-----

(i) $0 \le \theta < 2\pi, 0 \le \phi \le \pi$

(ii) $-\pi \le \theta \le \pi, 0 \le \phi \le \pi$

(iii) $-\frac{\pi}{2} \le \theta \le {}^{\pi}/_{2}, 0 \le \phi < 2\pi$

(iv) $0 < \theta \le \pi, -\pi \le \phi < \pi$

Vector Calculus Formula

SL. No	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
(a)	Differential Displacement $dl = dx\, a_x + dy\, a_y + dz\, a_z$	$dl = d\rho a_\rho + \rho d\phi a_\phi + dz a_z$	$dl = dr a_r + r d\theta a_\theta + r \sin \theta d\phi a_\phi$
(b)	Differential Area $dS = dy\, dz\, a_x$ $= dx\, dz\, a_y$ $= dx\, dy\, a_z$	$dS = \rho\, d\phi\, dz\, a_\rho$ $= d\,\rho\, dz\, a_\phi$ $= \rho d\rho d\,\phi\;\; a_z$	$d_s = r^2 \sin\theta\, d\theta\, d\phi\, a_r$ $= r \sin\theta\, dr\, d\phi\, a_\theta$ $= r\, dr\, d\theta\, a_\phi$
(c)	Differential Volume $dv = dx\, dy\, dz$	$dv = \rho\, d\rho\, d\phi\, dz$	$dv = r^2 \sin\theta\, d\theta\, d\phi\, dr$

Operators

1) ∇V – Gradient, of a Scalar V
2) $\nabla \overline{V}$ – Divergence, of a Vector $\overline{V}$
3) $\nabla \times \overline{V}$ – Curl, of a Vector $\overline{V}$
4) $\nabla^2 V$ – Laplacian, of a Scalar V

DEL Operator:

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z}\ a_z (\text{Cartesian})$$

$$= \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z}\ a_z (\text{Cylindrical})$$

$$= \frac{\partial}{\partial r} a_r + \frac{1}{r}\frac{\partial}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\ \frac{\partial}{\partial \phi}\ a_\phi (\text{Spherical})$$

Gradient of a Scalar field

V is a vector that represents both the magnitude and the direction of maximum space rate of increase of V.

$$\nabla V\ = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z}\ a_z \text{ For Cartisian Coordinates}$$

$$= \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z}\ a_z \text{ For Spherical Coordinates}$$

$$= \frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\ \frac{\partial V}{\partial \phi}\ a_\phi \text{ For Cylindrical Coordinates}$$

The following are the fundamental properties of the gradient of a scalar field V

1. The magnitude of ∇V equals the maximum rate of change in V per unit distance.
2. ∇V points in the direction of the maximum rate of change in V.
3. ∇V at any point is perpendicular to the constant V surface that passes through that point.
4. If $A = \nabla V$, V is said to be the scalar potential of A.
5. The projection of ∇V in the direction of a unit vector a is ∇V. a and is called the directional derivative of V along a. This is the rate of change of V in direction of a.

Example: Find the Gradient of the following scalar fields:

(a) $V\ = e^{-z} \sin 2x \cosh y$
(b) $U\ = \rho^2 z \cos 2\phi$
(c) $W = 10r \sin^2\theta \cos\phi$

Solution:

(a) $\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$

$= 2e^{-z} \cos 2x \cosh y\ a_x + e^{-z} \sin 2x \sinh y\ a_y - e^{-z} \sin 2x \cosh y\ a_z$

(b) $\nabla U = \frac{\partial U}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial U}{\partial \phi} a_\phi + \frac{\partial U}{\partial z} a_z$

$= 2\rho z \cos 2\phi\ a_\rho - 2\rho z \sin 2\phi\ a_\phi + \rho^2 \cos 2\ \phi\ a_z$

(c) $\nabla W = \frac{\partial W}{\partial r} a_r + \frac{1}{r}\frac{\partial W}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial W}{\partial \phi} a_\phi$

$= 10 \sin^2\theta \cos\phi\ a_r + 10 \sin 2\theta \cos\phi\ \ a_\theta - 10 \sin\theta\ \sin\phi\ a\phi$

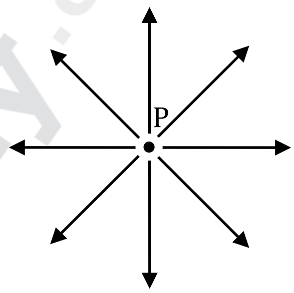
Divergence of a Vector

Statement: Divergence of $\overline{A}$ at a given point P is the outward flux per unit volume as the volume shrinks about P.

Hence,

$$\text{Div}A = \nabla . A = \lim_{\Delta v \to 0} \frac{\oint_S A . ds}{\Delta v} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where, Δv is the volume enclosed by the closed surface S in which P is located. Physically, we may regard the divergence of the vector field A at a given point as a measure of how much the field diverges or emanates from that point.



$$\nabla . A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}\frac{\partial A_z}{\partial z} \quad \text{Cartisian System}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad \text{Cylindrical System}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi} \quad \text{Sphearical System}$$

From equation (1),

$$\oint_S A . dS = \int_V \nabla . A \, dv$$

This is called divergence theorem which states that the total outward flux of the vector field A through a closed surface S is same as the volume integral of the divergence of A.

Example: Determine the divergence of these vector field

(a) $P = x^2 yz a_x + xz a_z$

(b) $Q = \rho \sin\phi \ a_\rho + \rho^2 z a_\phi + z\cos\phi \ a_z$

(c) $T = \frac{1}{r^2}\cos\theta \ a_r + r\sin\theta\cos\phi \ a_\theta + \cos\theta \ a_\phi$

Solution:

(a) $\nabla . P = \frac{\partial}{\partial x}P_x + \frac{\partial}{\partial y}P_y + \frac{\partial}{\partial z}P_z$

$$= \frac{\partial}{\partial x}(x^2 yz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(xz)$$

$$= 2xyz + x$$

(b) $\nabla . Q = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho Q_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \phi}Q_\phi + \frac{\partial}{\partial z}Q_z$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^2 \sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(\rho^2 z) + \frac{\partial}{\partial z}(z\cos\phi)$$

$$= 2\sin\phi + \cos\phi$$

(c) $\nabla . T = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(T_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(T_\phi)$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(r\sin^2\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\cos\theta)$$

$$= 0 + \frac{1}{r\sin\theta}2r\sin\theta\cos\theta\cos\phi + 0$$

$$= 2\cos\theta\cos\phi$$

Curl of a Vector

Curl of a Vector field provides the maximum value of the circulation of the field per unit area and indicates the direction along which this maximum value occurs.

That is,

$$\text{Curl } A = \nabla \times A = \lim_{\Delta S \to 0} \left(\frac{\oint_L A \,.\, dl}{\Delta S} \right)_{max} a_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \frac{1}{\rho} \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_\rho & r a_\theta & r \sin\theta \, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin\theta \, A_\phi \end{vmatrix}$$

From equation (2) we may expect that

$$\oint_L \mathbf{A\, dl} = \int_S (\nabla \times \mathbf{A}) \,.\, \mathbf{ds}$$

This is called stoke's theorem, which states that the circulation of a vector field A around a (closed) path L is equal to the surface integral of the curl of A over the open surface S bounded by L, Provided A and $\mathbf{\Delta} \times \mathbf{A}$ are continuous no s.

Example: Determine the curl of each of the vector fields.

(a) $\mathbf{P = x^2yz\, a_x + xza_z}$

(b) $\mathbf{Q = \rho \sin\phi a_\rho + \rho^2\, za_\phi + z \cos\phi a_z}$

(c) $\mathbf{T = \frac{1}{r^2} \cos\theta\ \ a_r + r\, \sin\theta \cos\phi a_\theta + \cos\phi\, a_\phi}$

Solution:

(a) $$\nabla \times \mathbf{P} = \left(\frac{\partial P_z}{\partial y} - \frac{\partial P_y}{\partial z}\right) a_x + \left(\frac{\partial P_x}{\partial z} - \frac{\partial P_z}{\partial x}\right) a_y + \left(\frac{\partial P_y}{\partial x} - \frac{\partial P_x}{\partial y}\right) a_z$$

$$= (0 - 0)a_x + (x^2y - z)a_y + (0 - x^2z)a_z$$

$$= (x^2y - z)a_y - x^2za_z$$

(b) $$\nabla \times \mathbf{Q} = \left[\frac{1}{\rho}\frac{\partial Q_z}{\partial \phi} - \frac{\partial Q_\phi}{\partial z}\right] a_\rho + \left[\frac{\partial Q_\rho}{\partial z} - \frac{\partial Q_z}{\partial \rho}\right] a_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho Q_\phi) - \frac{\partial Q_\rho}{\partial \phi}\right] a_z$$

$$= \left(\frac{-z}{\rho} \sin\phi - \rho^2\right) a_\rho + (0 - 0)a_\phi + \frac{1}{\rho}(3\rho^2 z - \rho \cos\phi)a_z$$

$$= -\frac{1}{\rho}(z \sin\phi + \rho^3)a_\rho + (3\rho z - \cos\phi)a_z$$

(c) $$\nabla \times \mathbf{T} = \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial \theta}(T_\phi \sin\theta) - \frac{\partial}{\partial \phi} T_\theta\right] a_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial \phi} T_r - \frac{\partial}{\partial r}(rT_\phi)\right] a_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rT_\theta) - \frac{\partial}{\partial \theta} T_r\right] a_\phi$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\cos\theta\sin\theta) - \frac{\partial}{\partial\phi}(r\sin\theta\cos\phi)\right]a_r$$
$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\frac{(\cos\theta)}{r^2} - \frac{\partial}{\partial r}(r\cos\theta)\right]a_\theta$$
$$+\frac{1}{r}\left[\frac{\partial}{\partial r}(r^2\sin\theta\cos\phi) - \frac{\partial}{\partial\theta}\frac{(\cos\theta)}{r^2}\right]a_\phi$$
$$= \frac{1}{r\sin\theta}(\cos 2\theta + r\sin\theta\sin\phi)a_r + \frac{1}{r}(0-\cos\theta)a_\theta$$
$$+\frac{1}{r}\left(2r\sin\theta\cos\phi + \frac{\sin\theta}{r^2}\right)a_\phi$$
$$= \left(\frac{\cos 2\theta}{r\sin\theta} + \sin\phi\right)a_r - \frac{\cos\theta}{r}a_\theta + \left(2\cos\phi + \frac{1}{r^3}\right)\sin\theta\ a_\phi$$

Laplacian

(a) Laplacian of a scalar field V, is the divergence of the gradient of V and is written as $\nabla^2 V$.

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \rightarrow \text{For Cartisian Coordinates}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2} \rightarrow \text{For Cylindrical Coordinates}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial\phi^2} \rightarrow \text{For Spherical Coordinates}$$

If $\nabla^2 V = 0$, V is said to be harmonic in the region.

A vector field is solenoid if $\nabla . A = 0$; it is irrotational or conservative if $\nabla \times A = 0$

$\nabla.(\nabla \times A) = 0$

$\nabla \times (\nabla V) = 0$

(b) Laplacian of Vector $\bar{A}$

$\nabla^2\vec{A} = \cdots$ is always a vector quantity

$\nabla^2\vec{A} = (\nabla^2 A_x)\hat{a}x + (\nabla^2 A_y)\hat{a}y + (\nabla^2 A_z)\hat{a}z$

$\nabla^2 A_x \rightarrow$ Scalar quantity

$\nabla^2 A_y \rightarrow$ Scalar quantity

$\nabla^2 A_z \rightarrow$ Scalar quantity

$\nabla^2 V = \frac{-p}{\epsilon}$........Poission's E.q.

$\nabla^2 V = 0$ ....Laplace E.q.

$\nabla^2 E = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu E\frac{\partial^2 E}{\partial t^2}$....... wave E. q.

Example: The potential (scalar) distribution in free space is given as $V = 10y^4 + 20x^3$.
If ε_0: permittivity of free space what is the charge density ρ at the point (2,0)?

Solution: Poission's Equation $\nabla^2 V = -\frac{\rho}{\varepsilon}$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(10\,y^4 + 20x^3) = \frac{-\rho}{\varepsilon_0}$$

$\because \varepsilon = \varepsilon_r\,\varepsilon_0[\varepsilon = \varepsilon_0 \text{ as } \varepsilon_r = 1]$

$20 \times 3 \times 2x + 10 \times 4 \times 3y^2 = \frac{-\rho}{\varepsilon_0}$

$$\text{At pt}(2,10) \Rightarrow 20 \times 3 \times 2 \times 2 = \frac{-\rho}{\varepsilon_0} \rho = -240\varepsilon_0$$

Example: Find the Laplacian of the following scalar fields

(a) $V = e^{-z} \sin 2x \cosh y$

(b) $U = \rho^2 z \cos 2\phi$

(c) $W = 10r \sin^2\theta \cos\phi$

Solution: The Laplacian in the Cartesian system can be found by taking the first derivative and later the second derivative.

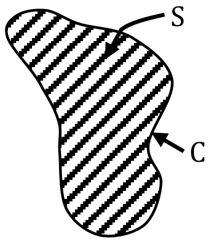
(a) $$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$
$$= \frac{\partial}{\partial x}(2e^{-z} \cos 2x \cosh y) + \frac{\partial}{\partial y}(e^{-z} \sin 2x \sinh y) + \frac{\partial}{\partial z}(-e^{-z} \sin 2x \cosh y)$$
$$= -4e^{-z} \sin 2x \cosh y + e^{-z} \sin 2x \cosh y + e^{-z} \sin 2x \cosh y$$
$$= -2e^{-z} \sin 2x \cosh y$$

(b) $$\nabla^2 U = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial U}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2}$$
$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(2\rho^2 z \cos 2\phi) - \frac{1}{\rho^2} 4\rho^2 z \cos 2\phi + 0$$
$$= 4z \cos 2\phi - 4z \cos 2\phi$$
$$= 0$$

(c) $$\nabla^2 W = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial W}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial W}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 W}{\partial \phi^2}$$
$$= \frac{1}{r^2}\frac{\partial}{\partial r}(10\, r^2 \sin^2\theta \cos\phi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(10r \sin 2\theta \sin\theta \cos\phi) - \frac{10r \sin^2\theta \cos\phi}{r^2 \sin^2\theta}$$
$$= \frac{20 \sin^2\theta \cos\phi}{r} + \frac{20r \cos 2\theta \sin\theta \cos\phi}{r^2 \sin\theta} + \frac{10r \sin 2\theta \cos\theta \cos\phi}{r^2 \sin\theta} - \frac{10 \cos\phi}{r}$$
$$= \frac{10 \cos\phi}{r}(2\sin^2\theta + 2\cos 2\theta + 2\cos^2\theta - 1)$$
$$= \frac{10 \cos\phi}{r}(1 + 2\cos 2\theta)$$

Stoke's Theorem

Statement: Closed line integral of any vector $\vec{A}$ integrated over any closed curve C is always equal to the surface integral of curl of vector $\vec{A}$ integrated over the surface area 's' which is enclosed by the closed curve 'c'.



$$\oint \vec{A}\cdot d\vec{L} = \int_S \int (\nabla \times \vec{A})\, d\vec{S}$$

The theorem is valid irrespective of

(i) Shape of closed curve 'C'